The properties of dark matter

Ernst Fischer
e.fischer.stolberg@t-online.de

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Abstract

Observations of density profiles of galaxies and clusters constrain the properties of dark matter. Formation of stable halos by collisional fluids with very low mass particles appears as the most probable interpretation, while halos formed by high mass particles, left over from a hot big bang, can scarcely explain the observed density distributions. Detection methods of dark matter are discussed.

1 Introduction

The fact that gravitational dynamics of the universe is dominated by some matter component, which cannot be detected by any electromagnetic signals, is one of the most discussed problems in cosmology. There are two very pronounced effects, which are ascribed to this dark matter. One is the observed high velocity of galaxies in clusters, which exceeds by far the escape velocity of the cluster, if only the masses of stars and gas are taken into account. The other is the constancy of the rotation velocity of spiral galaxies in the outer regions, where observed stars and gas contribute only little to the matter budget.

Various ideas have been discussed to explain, what this dark matter may be, beginning with weakly interacting particles, left over from a hot beginning of the universe (WIMPs), new kinds of neutrinos or other very light particles, proposed by some theories of grand unification (GUT) or supersymmetric partners to our known particles, proposed by string theory. Even the influence of particles living in extra dimensions has been invoked. Rather comprehensive overviews on these ideas have been given by Ostriker and Steinhardt [1] and more recently by Feng [2].

Most of the papers in the field are inspired by concepts, which relate the existence of dark matter to effects, which took place in a very hot beginning of the universe and thus to the physical processes, which may have occurred during the cooling process from a state of extreme energy density. Depending on the physical model of the 'big bang' process and on the underlaying models of high energy particle physics, various particle candidates are discussed and investigated with respect to their ability to act as dark matter.

But all these theoretical models are more or less speculative and we have no definite proofs of their correctness. Thus in this paper we will not follow the usual way, but focus first on the question, what we can learn from present observations on the properties of the dark matter particles and only after that look for possible candidates, to meet these requirements. From the fact that any good theory of particle physics must supply us with the corresponding particles we can finally decide on the best choice or even require a revision of all existing models.

2 Properties of dark matter

From observations of galaxy clusters only rather vague conclusions can be drawn on the properties of dark matter particles. The fact that apart from their influence on the gravitational balance we see no measurable effects is only suitable to exclude possible properties. The most obvious statement, which one can make, is that dark matter does non interact electromagnetically. It has no electrical charge and cannot decay into charged particle pairs or photons. Such events would be detectible as some characteristic form of radiation.

The same argument holds for interactions with ordinary matter. Collisions with neutral atoms would lead to excitation or ionization, if the collisional energy is high enough. The absence of such processes excludes dark matter particles with high collisional energy, be it nonrelativistic particles with very high mass or relativistic particles with low mass. These arguments have led many authors to the conclusion that dark matter is 'collisionless'. But this can only refer to inelastic or reactive collisions.

Elastic interactions with energy transfer far below the excitation levels of atoms or molecules are possible, of course, as well as interactions between dark matter particles. These interactions may be direct collisions or momentum exchange in the gravitational field of neighboring particles. Without an exchange of energy and momentum dark matter particles could never be captured by existing matter concentrations. They would be gravitationally accelerated towards the center and after the passage would fly out again without leaving any trace.

Thus observation of the rotation curves of spiral galaxies adds an additional constraint to the properties of dark matter particles. The constancy of the rotation velocity in the outer regions, where dark matter dominates the mass distribution, requires a matter density profile decreasing with distance from the center as $1/r^2$. The radial dependence of the dark matter density can only be understood, if these particles transfer energy and momentum between one another, so that the energy which they take up from the gravitational field can be transferred from radial into thermal motion.

These constraints show that dark matter behaves just like an ideal gas under the influence of gravitation, but the kinetic energy of the individual dark matter particles must be less than the excitation energy of ordinary atoms or molecules. Such an ideal gas of low mass particles and with low energy density

would scarcely be able to condense into closed structures like galaxies or galaxy clusters, as the energy gained from the gravitational field would build up a pressure, which would counteract further concentration. Instead fluctuations would develop into a web-like structure as we observe it in the large scale structure of the universe and which can be well reproduced by numerical simulations. But in contrast, ordinary matter which can lose energy by electromagnetic radiation, may form very concentrated structures by gravitational instability, which then can attract dark matter from an ubiquitous homogeneous distribution. By a self-enhancing process this dark matter concentrates into a state, where it dominates over ordinary matter.

3 Modelling dark matter distributions

The fact that dark matter halos are found as well around galaxy clusters as around individual galaxies, be it small or large, be it old or young, must be regarded as a hint that these halos are stable equilibrium configurations, at least on a scale of galaxy life times, and not relics of an early epoch of the universe. Thus as a first approximation we can regard them as assemblies of some kind of ideal gas, which is in virial equilibrium. At least in the outer regions the influence of ordinary matter to the equilibrium conditions can be neglected, though condensations of ordinary matter should be regarded as the primary source of structure formation.

Apparently dark matter particles feel no other force than gravitation and can exchange kinetic energy or momentum only by elastic gravitational interaction or by direct collisions. But as will be discussed later, the contribution of distant interactions appears negligible, as the mean free path is too large to establish equilibrium in galaxy sized objects. The energy which the particles take up in a gravitational field is transferred to thermal motion preferably by collisions. Whether this thermal energy remains in the region, where it is produced, or if it is transported by conduction, depends on the mean free path. In the limit of small mean free path the local energy density equals the potential energy taken up during infall into the halo, the normal virial condition

$$2E + U = 2E_0 + U_0, \tag{1}$$

where E and U are the density of kinetic and potential energy and E_0 and U_0 the respective values outside the halo. Equilibrium is obtained when the local energy is redistributed homogeneously into all degrees of freedom yielding an homogeneous pressure $p = -U/3$. The dynamic equilibrium condition in a radially symmetric halo can by described by

$$\frac{dp}{dr} = -\varrho \frac{d\Phi}{dr}, \tag{2}$$

where Φ is the gravitational potential, defined in such a way that it vanishes outside the halo. Under the assumption that ordinary matter with mass M_0 is

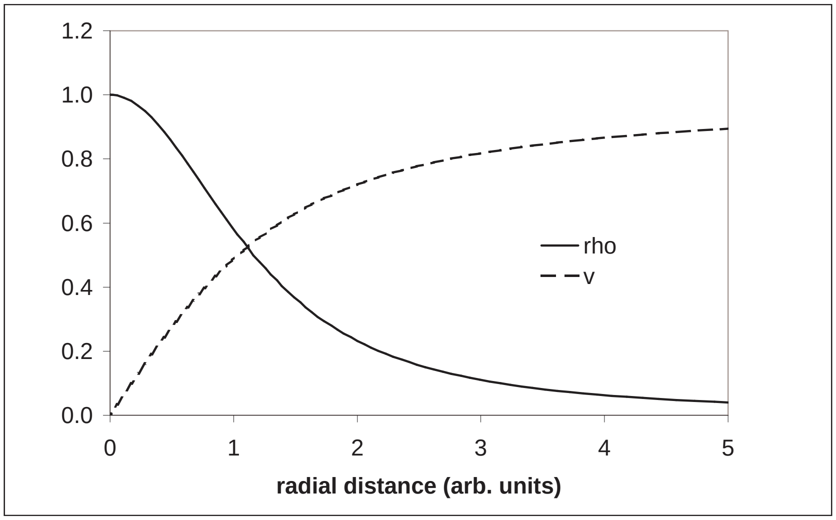

Figure 1: Solid line: relative radial density distribution, dashed line: circular equilibrium velocity

concentrated inside some radius r_0, outside this radius the potential is given by

$$\Phi - \Phi_0 = \frac{G}{r}\left(M_0 + 4\pi \int_{r_0}^{r} \varrho\, r'^2 dr'\right), \tag{3}$$

where Φ_0 has to be chosen so that $\Phi = 0$ at infinity, assuming that the dark matter has zero energy outside the halo. Thus with the ideal gas condition $p = 2/3E$ and $U = -\varrho\Phi$ eq.(2) reads

$$\frac{dp}{dr} = -\frac{1}{3}\frac{d(\varrho\,\Phi)}{dr} = -\varrho\frac{d\Phi}{dr}. \tag{4}$$

with the general solution $\varrho = C\Phi^2$. Together with the relation between the potential and the matter density eq.(3) this leads to the result that the potential approaches zero as $1/r$ and the matter density decreases as $1/r^2$. In the regions, where the density of ordinary matter is negligible, the normalized radial profile takes the simple analytical form

$$\varrho = (\tanh(r)/r)^2, \tag{5}$$

which is valid up to the region, where the influence of other structures becomes important or where the time of formation is too long to reach equilibrium. Fig.1 shows the general form of the density profile together with the expected profile of the equilibrium circular velocity v, defined by $\varrho v^2/r = \varrho\frac{d\Phi}{dr}$. The absolute value of the dark matter density remains undefined from this model. It is determined by the total amount available in the corresponding region of space, which will be distributed among the ordinary matter concentrations like galaxies during their period of formation.

It should be stressed here that the density profile of dark matter is quite different from the frequently used NFW profile (Navarro, Frenk and White [3]), but agrees much better with observations, as well with respect to the asymptotic power law $1/r^2$ as to the flat core distribution.

By now we have demonstrated that the most probable candidate of dark matter is an ideal gas, which permeates all space and condenses onto structures, which are formed by ordinary matter. The dark matter particles do not exhibit any electromagnetic interaction, but exchange energy and momentum only by direct collisions and by gravity.

We can obtain further information on the nature of these particles from the fact that, while collisions between them lead to thermalization on the scale of galaxies, particles like protons or nuclei, which we observe as cosmic rays, transverse large parts or even the entire galaxy without considerable loss of energy. That means that the number of collisions must be high, but the energy transfer to protons or other nuclei is negligible. The mass of the dark matter particles must be very small compared to the proton mass.

It has been argued that dark matter should be collisionless (Markevitch et al.[4]), because observations of two merging galaxy clusters, the so called bullet cluster, show that in determinations of the total matter distribution the dark matter appears to follow the collisionless motion of galaxies, but the hot x-ray emitting cluster gas is displaced due to ram pressure. But this does only show that the dark matter is coupled to the individual galaxies like a fixed atmosphere, unlike the intracluster plasma, which appears continuously distributed all over the cluster volume. Every galaxy is dressed by its own dark matter halo. Thus the different behavior of the dark matter and the hot plasma can be regarded as an additional hint that dark matter is highly collisional and thermalized, so that it follows the motions of concentrations of ordinary matter in galaxies.

We can try to get an idea on the mass of dark matter particles from the fact that the halos are thermalized on the scale of galaxies, that means that the mean free path of the particles is much less than the characteristic length of the matter distribution.

That distant gravitational interactions are negligible, can easily be estimated. We consider the motion of a particle with initial velocity v in a medium of density ϱ consisting of particles of mass m, which is deflected by the gravitational attraction of other particles. We put the question, how far it must travel until a series of such small deflections adds up to a change of the direction of motion, so that the perpendicular velocity equals the the initial velocity $v_\perp = v$. The number density of particles is $n = \varrho/m$, their mean distance $d = 1/n^{1/3}$. The impact parameter b determines the deflection of the particle trajectory. The force perpendicular to v at distance $r = \sqrt{x^2 + b^2}$ is

$$F = \frac{Gm^2 b}{r^3} \tag{6}$$

(G is the gravity constant). Integrating from infinity to the point of closest approach leads to a transverse velocity $v_\perp = F/2m$ (both particles are deflected

in opposite directions)

$$v_\perp = \int_{-\infty}^{b} \frac{Gmb}{2vr^2\sqrt{r^2-b^2}}dr = \frac{Gm}{2bv} \tag{7}$$

Averaging over all possible impact parameters from 0 to $d/2$ yields

$$v_\perp = \frac{4}{\pi d^2}\int_0^{d/2} \frac{Gm}{2bv} 2\pi b\, db = \frac{2Gm}{dv} \tag{8}$$

The average perpendicular velocity after N deflections of statistical direction is $\frac{2Gm}{dv}\sqrt{N}$. The mean free path λ is reached when $v_\perp = v$.

$$\lambda = \frac{Nd}{2} = \frac{1}{2}\sqrt{\frac{d^3v^2}{2Gm}} = \sqrt{\frac{v^2}{2Gmn}} = \sqrt{\frac{v^2}{2G\varrho}} \tag{9}$$

independent of the size of the individual particles.

In a galaxy, where the density profile can be approximated by $\varrho = A/r^2$ and the equilibrium condition $v^2/r = GM/r^2$ holds, this leads to the invariant condition

$$\lambda = \sqrt{\frac{\pi}{2}}r \tag{10}$$

That means that the mean free path is in the order of the characteristic length of the matter distribution. This condition is based on the same physics as underlying the Jeans criterion of gravitational instability. This gravitational interaction may be responsible for the formation of large scale structure in the universe, but to establish thermal equilibrium in the interior of galaxy halos, much shorter mean free path is required.

There must be additional interactions between dark matter particles. Interaction is not restricted to the mutual gravitational attraction, but there must be also direct collisions. As we take for sure that dark matter particles obey some form of quantum physics, they must be regarded as fields with a spatial extension of their wave function in the order of the Compton wavelength. Consequently we can expect that there exist interactions, when the distance between particles is less than this length scale. The collision cross section thus is of the order

$$\sigma = \left(\frac{hc}{\varepsilon}\right)^2 \tag{11}$$

where ε is the rest energy of the particles. The number density of particles in a region of total energy density E is $n = E/\varepsilon$, so that we obtain the mean free path

$$\lambda = \frac{1}{\sigma n} = \frac{\varepsilon^3}{(hc)^2 E}. \tag{12}$$

In a galaxy this length must be small compared to that of the gravitational interactions mentioned before. Let us take the conditions in the solar system as

a typical example. The rest energy density can be calculated from the rotation velocity of the galaxy in our neighborhood $v = 220$ km/s. Assuming a spherical matter distribution and a total density profile with slope $1/r^2$, at the distance $r_0 = 6.7$ kpc from the center we find an energy density

$$E = \frac{v^2 c^2}{4\pi G r_0^2} = 0.76 \text{ GeV}. \tag{13}$$

More detailed density profiles, taking additionally into account stellar matter and gas distribution, give similar results, all in the order of 1 GeV. The upper limit of the mean free path, to achieve local equilibrium must be small compared to the local characteristic length of the density gradient, that means, at most of the order $\lambda = 1$ kpc. Thus from eq.(12) we find $\varepsilon < 36$ MeV. This rules out as constituents of dark matter any of the GeV or TeV particles, proposed from supersymmetry or similar elementary particle models.

Unfortunately we cannot derive any lower bound from such estimations. The only thing we can learn from eq.(12) is that the mean free path changes with the third power of the particle rest energy. Energies in the order of a few eV, as they are discussed as rest energies of neutrinos, would result in a mean free path in the order of cm. This would also rule out neutrinos as possible candidates, as we observe neutrinos travelling deep into the earth without considerable attenuation. But we must keep in mind that our estimations are valid only for thermal particles. Relativistic particles may exhibit interaction cross sections many orders of magnitude lower.

There may exist other particle species, which are unknown to us by now, though they exist everywhere in the solar system and even on earth. But due to their extremely weak interaction with ordinary matter they have escaped detection by now. If these particles interact only gravitationally, they do not even 'know' what electromagnetism is. Thus interactions with photons are excluded, and even annihilation processes with the corresponding antiparticles would be impossible, as there is no resulting photon pair to carry the energy away. Such particles may exist as a stable mixture of particles and antiparticles (or they are their own antiparticles). The only way to change their number would be annihilation by three body interactions and pair creation by collisions in the extreme tail of their kinetic energy distribution.

4 Detection of dark matter particles

From the properties of dark matter particles, which we have discussed in the last section, it appears difficult to detect individual particles directly. We expect that due to their low mass and low kinetic energy reactive collisions with ordinary matter are extremely improbable. All the experiments focussed to the detection of GeV or TeV particles will scarcely obtain positive results. They may find some high energy events. But these should not be attributed to the dark matter, which forms the halos around galaxies. Instead they must be caused by processes in active galaxy cores or similar high energy environments.

As we assume that the kinetic energy of dark matter particles in our environment is too low to react with ordinary matter, the only way to obtain measurable interactions is, to supply the collision energy by highly accelerated ordinary matter. There are sufficient numbers of dark particles everywhere around us. From the rest energy density of about $1\,\mathrm{GeV/cm^3}$ in the solar neighborhood we expect 10^3 particles per cm^3 with 1 MeV rest energy or 10^9 particles of 1eV.

Thus the best chance to observe reactions of these particles is in accelerators like Tevatron or LHC, and it may well be that we already have many such observations without considering them as influenced by dark matter particles. Collisions in these accelerators take place not in absolute vacuum, but in a medium filled with the ubiquitous dark matter, which permeates the vacuum tubes unimpeded. Thus collisions between the accelerated ordinary matter particles may by chance take place in the presence of a dark particle, which can change the balance of quantum numbers. Observed violations of CP invariance in weak interactions or flavor non-conservation may well be the result of interference with the dark matter particles.

It has to be proved, if a systematic investigation of these processes, which violate established quantum rules, can be explained by interactions with dark matter. There is some hope that it might be possible to extract additional information from archived data, when they are examined in the light of this new interpretation.

Of course, these ideas are somewhat speculative, but we should keep them as a possible alternative, at least as we have no observations conflicting with this concept. By now we have no positive identification of dark matter particles, but from observations we can rule out with high probability all the models, which are based on some high mass particles, left over from a hot big bang.

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